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1992 J. Phys. A: Math. Gen. 25 2417

(http://iopscience.iop.org/0305-4470/25/8/048)

J. Phys. A: Math. Gen. **25** (1992) 2417–2418. Printed in the UK

COMMENT

Comment on 'Towards the conservation laws and Lie symmetries for the Khokhlov–Zabolotskaya equation in three dimensions'

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Received 25 November 1991

Abstract. We comment on the Lie point symmetries for the Khokholov-Zabolotskaya equation as calculated by Roy Chowdhury and Nasker, and demonstrate that their result for the coefficients of the vector field is correct but incomplete.

The Lie point symmetries for the Khokhlov–Zabolotskaya equation

$$\rho_{xt} - (\rho\rho_x)_x = \rho_{yy} \tag{1}$$

were calculated by Roy Chowdhury and Nasker [1]. Following their notation they considered the vector field

$$\alpha = \eta_1 \frac{\partial}{\partial\rho} + \eta_2 \frac{\partial}{\partial x} + \eta_3 \frac{\partial}{\partial t} + \eta_4 \frac{\partial}{\partial y} \tag{2}$$

and determined the coefficients to be

$$\eta_1 = (a-b)\rho + cy + d \tag{3}$$

$$\eta_2 = ax - (cy+d)t + e \tag{4}$$

$$\eta_3 = bt + e \tag{5}$$

$$\eta_4 = \tfrac{1}{2}(a+b)y - ct^2 + e \tag{6}$$

where a, b, c, d and e are all constants. Apart from the missing factor t in η_2, most likely a misprint in their equation (7), this result is correct but incomplete.

The authors recalculated the Lie point symmetries [2] of (1) with the program SYMMGRP.MAX developed for this purpose by Champagne *et al* [3]. The coefficients of the vector field are in their most general form given by

$$\eta_1 = \rho(k - \tfrac{2}{3}f') - \tfrac{1}{3}xf'' - \tfrac{1}{2}yh'' - \tfrac{1}{6}y^2 f''' - g' \tag{7}$$

$$\eta_2 = x(k + \tfrac{1}{3}f') + \tfrac{1}{2}yh' + \tfrac{1}{6}y^2 f'' + g \tag{8}$$

$$\eta_3 = f \tag{9}$$

$$\eta_4 = \tfrac{1}{2}y(k + \tfrac{4}{3}f') + h \tag{10}$$

where f, g and h are arbitrary smooth functions of t only, and k is an arbitrary constant.

The corresponding infinitesimal generators for the Khokhlov–Zabolotskaya equation are then

$$G_1 = -\tfrac{1}{3}(2\rho f' + xf'' + \tfrac{1}{2}y^2 f''')\partial_\rho + \tfrac{1}{6}(2xf' + y^2 f'')\partial_x + f\partial_t + \tfrac{2}{3}yf'\partial_y \tag{11}$$

$$G_2 = -g'\partial_\rho + g\partial_x \tag{12}$$

$$G_3 = -\tfrac{1}{2}yh''\partial_\rho + \tfrac{1}{2}yh'\partial_x + h\partial_y \tag{13}$$

$$G_4 = \rho\partial_\rho + x\partial_x + \tfrac{1}{2}y\partial_y. \tag{14}$$

The very special case for the coefficients (3)–(6) considered by Roy Chowdhury and Nasker [1] follows from (7)–(10) by selecting

$$f = bt + e \tag{15}$$

$$g = -dt + e \tag{16}$$

$$h = -ct^2 + e \tag{17}$$

$$k = a - \tfrac{1}{3}b. \tag{18}$$

References

[1] Roy Chowdhury A and Nasker M 1986 *J. Phys. A: Math. Gen.* **19** 1775–81

[2] Euler N and Steeb W-H 1992 *Continuous Symmetries, Lie Algebras and Differential Equations* (Mannheim: Bibliographisches Institut)

[3] Champagne B, Hereman W and Winternitz P 1991 *Comput. Phys. Commun.* **66** 319–40